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TRANSDUCER CALIBRATION FROM NEAR-FIELD DATA

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## ABSTRACT

The theory for obtaining free-field sensitivities and transmitting responses of line or piston-type transducers from near-field data and the equations for obtaining the directivity, transmitting or receiving, from near-field data are derived. The set-up and adjustment of the Scientific-Atlanta Corporation type CF4 Fourier Integral Computer for computing the data are described and the data that must be obtained are listed.

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# LETTER SYMBOLS

$r_0$	radius of a circular piston, or half the length of the side of a square piston source
$k$	$= 2\pi/\lambda$ , where $\lambda$ is the wavelength of the sound
$L$	length of a line source
$M_p$	free-field voltage sensitivity of a piston transducer
$S_p$	plane-wave near-field transmitting current response of a piston transducer
$S_{Sp}$	spherical-wave far-field transmitting current response of a piston transducer
$M_L$	free-field voltage sensitivity of a line transducer
$S_{CL}$	cylindrical-wave near-field transmitting current response of a line transducer
$S_{SL}$	spherical-wave far-field transmitting current response of a line transducer
$M_T$	free-field voltage sensitivity of the measuring transducer
$S_T$	spherical-wave far-field transmitting current response of the measuring transducer
$a$	reference distance for $S_p$ or $S_{CL}$ , the distance from the piston to the plane aperture or the distance from the line transducer to the line aperture in measurements of the near-field of a line transducer
$d$	reference distance for $S_{Sp}$ or $S_{SL}$ , generally 1 meter
$J_p(a)$	$= 2A/\rho c$ , the plane-wave reciprocity parameter for a piston transducer of area $A$
$J_S$	$= 2d\lambda/\rho c$ , the spherical-wave reciprocity parameter
$J_{CL}$	$= 2L(a\lambda)^{1/2}/\rho c$ , the cylindrical-wave reciprocity parameter for a line transducer of length $L$
$E_T, E_p, E_L$	open-circuit voltage of the measuring transducer, piston, and line transducer, respectively
$I_T, I_p, I_L$	driving current
$\psi$	phase between open-circuit voltage of a receiving transducer and the driving current of a source transducer in an measurement combination

# TRANSDUCER CALIBRATION FROM NEAR-FIELD DATA

## INTRODUCTION

The near sound field of a directional piston source is nondivergent and the wave is essentially plane. The wave impedance is  $\rho c$  for values of  $kr_0 > 5$ , where  $k = 2\pi/\lambda$  and  $r_0$  is the radius of a circular piston, half the length of a side for a square piston, or half the width of a rectangular piston. In this region, the plane-wave reciprocity parameter is applicable to calculations based on the average pressure over a plane aperture. The theory and application of the plane-wave reciprocity parameter have been described by Simmons and Urlick [1].

The wave in the near field of a line transducer is cylindrical, and the restriction on the line length is similar to that for the plane wave:  $k(L/2) > 5$ , where  $L$  is the length of the line. In this region, the cylindrical-wave reciprocity parameter is applicable to calculations based on the average pressure along a line aperture parallel to the transducer. Bobber and Sabin [2] have described the theory and application of the cylindrical-wave reciprocity parameter.

Stenzel [3] shows the radiation impedance for a circular and a square piston as a function of  $kr_0$ . The minimum line length stated above for cylindrical-wave reciprocity is based on the radiation impedance of a line element of a square piston that would produce a cylindrical wave without divergence at the ends of the line in a plane containing the line.

The relation between near-field transmitting response and its reciprocity parameter and that between the far-field transmitting response and the spherical-wave reciprocity parameter can be used to compute the near-field transmitting response and free-field receiving sensitivity from near-field data.

Far-field directivity can also be calculated from near-field data.

## THEORY

A probe or line transducer can be used to scan in detail the near sound field pressure of a piston transducer over a plane aperture. The measuring line transducer can be used to integrate, in one coordinate, to obtain the average pressure along its length. The measuring line transducer is then moved at right angles to its length to scan the near field of the piston transducer. The output voltage and relative phases of the measuring line transducer as a function of position can be used to obtain a line source equivalent to the piston transducer in any desired plane of directivity.

The free-field voltage sensitivity of the piston transducer is related to its near-field plane-wave transmitting current response and its far-field transmitting response by the plane-wave reciprocity parameter and the spherical-wave reciprocity parameter, respectively:

$$M_P = S_P(a)J_P(a) = S_{SP}(d)J_S(d). \quad (1)$$

The near-field transmitting current response is the average pressure over the plane aperture of area  $A$  in the near field divided by the current  $I_P$  driving the piston transducer whose active face also has an area  $A$ . The average pressure is obtained from the free-field voltage sensitivity of the probe or the unshaded measuring line transducer  $M_T$  and the integration of the open-circuit rms voltage  $E_T$  of the probe or line as a function of the position in the plane aperture at the distance  $a$  from the piston transducer. The voltage and its phase  $\psi$  relative to the current  $I_P$  are measured.

$$S_P(a) = \int E_T e^{-i\psi} dA / I_P M_T A, \quad a < A/\lambda. \quad (2)$$

The spherical-wave far-field transmitting current response from equations (1) and (2) is

$$\begin{aligned} S_{SP}(d) &= S_P(a)J_P(a)/J_S(d) \\ &= \frac{\int E_T e^{-i\psi} dA}{I_P M_T A} \frac{2A}{\rho c} \frac{\rho c}{2d\lambda} \\ &= \frac{\int E_T e^{-i\psi} dA}{I_P M_T} \frac{1}{d\lambda}. \end{aligned} \quad (3)$$

The ratio  $E_T/I_P = \pm E_P/I_T$  is the transfer impedance between the piston transducer and the measuring probe or line transducer for each and every point in the plane aperture. For a transducer combination in which the electromechanical coupling is electrostatic in one and electromagnetic in the other, the negative sign applies.

Thus, except for a possible change in sign,

$$\frac{\int E_T e^{-i\psi} dA}{I_P} = \frac{\int E_P e^{-i\psi} dA}{I_T}. \quad (4)$$

From equations (1), (3), and (4),

$$M_P = \frac{\int E_P e^{-i\psi} dA}{I_T S_T} \frac{1}{d\lambda}, \quad (5)$$



where  $S_T$  is the spherical-wave far-field transmitting current response of the probe or line transducer at the reference distance  $d$ .

Thus, by means of voltage and current measurements obtained in the near field over a plane aperture and the free-field voltage sensitivity of the measuring probe or line transducer, the spherical-wave far-field transmitting current response of a piston transducer can be obtained. Similar measurements with the spherical-wave far-field transmitting current response of the probe or line transducer will yield the free-field voltage sensitivity of a piston transducer.

The free-field voltage sensitivity of a line transducer is related to its near-field cylindrical-wave transmitting current response and far-field transmitting current response by the cylindrical and spherical-wave reciprocity parameters, respectively:

$$M_L = S_{CL}(a)J_C(a) = S_{SL}(d)J_S(d). \quad (6)$$

By the same argument as before,

$$S_{SL}(d) = \frac{\int_0^L E_T e^{-i\psi(x)} dx}{I_L M_T} \frac{(a\lambda)^{\frac{1}{2}}}{d\lambda} \quad (7)$$

and

$$M_L = \frac{\int_0^L E_L e^{-i\psi(x)} dx}{I_T S_T} \frac{(a\lambda)^{\frac{1}{2}}}{d\lambda}. \quad (8)$$

Reference /2/ gives the limits for the measuring distance  $a$  as  $3 < ka < 2\pi(L/\lambda)^2$ .

The same procedure can be used to derive the free-field current sensitivity and far-field transmitting voltage response, or these can be obtained from the given equations together with the transducer impedance.

#### COMPUTATION OF DATA

The piston transducer can be resolved into an equivalent line source in the plane for which the directivity is desired. The integration normal to this line in a plane parallel to the face of the piston can be performed by measuring in the plane with a line transducer. The expression for computing the directivity of a piston or a line transducer has been derived many times (see, for example, reference [4]) and can be expressed in terms of the open-circuit voltage output of a probe hydrophone in the near field of a line transducer, or the open-circuit voltage of a line hydrophone in the near field of a piston, together with the phase of the voltage referred to the phase of the measured transducer's driving current or voltage. Let  $E(x)$  be the voltage,  $\psi(x)$  the phase (leading phase is positive), and  $l$  the length

of traverse of the measuring transducer; then the directivity is given by

$$\frac{p(\phi)}{p(\phi=0)} = (1/2L) \int_0^L (1 + \cos \phi) e^{-i[kx \sin \phi + \psi(x)]} E(x) dx. \quad (9)$$

If the sound energy flow is not normal to the plane aperture but is a parallel flux (plane wave), then the factor 1 in equation (9) is replaced by the  $\cos \theta$ , where  $\theta$  is the angle between the normal to the aperture and the flux. Thus, for a transducer producing a depressed beam, the plane scanned in the near field can be the plane of the piston, and a correction  $\cos \theta$  inserted instead of 1. This correction is a potentiometer adjustment in the Model CF4 Fourier Integral Computer.

#### APPLICATION OF CF4 FOURIER INTEGRAL COMPUTER TO THE PROBLEM

The CF4 Fourier Integral Computer computes and plots directivity in the far field of a source from near-field data. The directivity is limited to 60 degrees on each side of the beam axis, or a total of 120 degrees.

If a probe hydrophone is used to measure the near field in a plane aperture of a piston transducer, the voltage output of the probe and its phase is a function of two coordinates  $x$  and  $y$ . The data are plotted in volts  $E(y)$  and in degrees  $\psi(y)$ , for each value of  $x$ . The voltage must come to zero at both ends of the graph. The plots of  $E(y)$  and  $\psi(y)$  are restricted to a 10 by 10 inch area by the size of the curve reader drums to which they are attached. The computer integrates  $E(y)$ ,  $\psi(y)$  for a value of  $x$  in eight seconds. From this computation, a plot is made of  $E(x)$  and  $\psi(x)$ , the integrated value in volts and its phase in degrees as read on the computer's oscilloscope. When a line hydrophone is used to perform the  $y$ -axis integration or when a probe hydrophone scans the line aperture of a line transducer, the operation just described is eliminated.

The plot of  $E(x)$  in volts is inserted in the upper curve reader, and the plot of  $\psi(x)$  is inserted in the lower curve reader. The computed directivity is plotted in decibels by a strip chart recorder in terms of a computer  $u$  scale. The  $u$  scale is proportional to  $\sin \phi$ , where  $\phi$  is the angle between the direction and the beam axis. Two adjustments, a phase argument switch and a potentiometer, establish the  $u$  scale of the computer for a particular plot of  $E(x)$  and  $\psi(x)$ . The two adjustments are  $k_1$  and  $k_2$ , where  $k_1$  is a potentiometer scale reading, and  $k_2$  is a phase argument switch with positions  $\pi = 1, 2, 4$  such that  $k_2 = \pi\pi/2$ . The  $u$  scale equation is  $k_1 k_2 u = (\pi\pi/\lambda) \sin \phi$ . The 10-inch drum length is equivalent to  $\pi$  inches in traverse of the probe or line transducer in the near field;  $\lambda$  is the wavelength of the sound;  $\phi$  is the azimuth angle of the computed directivity. The  $(1 + \cos \phi)$  term in equation (9) is fixed with relation to the  $u$  scale such that for  $u = 10$ , the azimuth angle  $\phi = 60^\circ$ . Thus

$$k_1 k_2 (10) = (\pi\pi/\lambda) \sin 60^\circ = (\pi\pi/\lambda)(\sqrt{3}/2)$$

and

$$k_1 = \frac{(\pi\pi/\lambda) (\sqrt{3}/2)}{10(\pi\pi/2)} = \sqrt{3} \pi / 10\pi\lambda.$$

In an example, a 30-inch scan was plotted on the 10-inch drum length, or  $n = 30$ . At 12.8 kc, the wavelength  $\lambda = 4.43$  inches. The phase argument switch was set at  $\pi = 1$ . Thus, the  $k_1$  potentiometer must be set at

$$k_1 = \frac{\sqrt{3}(30)}{10(1)(4.43)} = 1.17.$$

The computed directivity is then transcribed from the strip chart in azimuth angles by the proportion

$$\phi = \sin^{-1} [(u/10) \sin 60^\circ] = \sin^{-1}(u\sqrt{3}/20).$$

One unit of the  $u$  scale is 2 inches on the strip chart. A polar recorder can be obtained for plotting the level in decibels as a function of azimuth angle directly from the computer.

To compute the transmitting current response or free-field voltage sensitivity of a transducer by means of the computer and equations (3), (5), (7), and (8), the integrated level for  $\phi = 0$  must be referred to the level integrated by the computer for an ideal reference transducer producing a constant amplitude and phase over its aperture in the near field. If, in equation (5), the voltage  $E_p = E_1$ , a constant, and  $\psi = 0$ , the free-field voltage sensitivity  $M_{p \text{ ref}}$  of the ideal transducer (a piston transducer scanned with a line transducer of length  $L$  over a scan of length  $l$ ) is

$$M_{p \text{ ref}} = \frac{E_1 A}{I_T S_T} \frac{1}{d\lambda}$$

$$A = Ll$$

$$M_p = \frac{E_1}{I_T S_T} \frac{A}{d\lambda} \frac{\int_0^l E_p(x) e^{-i\psi(x)} dx}{\int_0^l E_1 dx},$$

where  $M_p$  is in volts per  $\mu\text{bar}$ ,  $S_T$  is in  $\mu\text{bars}$  per ampere,  $E_1$  is in volts, and  $I_T$  is in amperes. The difference in decibels between the two integrals is obtained from

the computer strip chart recorder for  $\phi = 0^\circ$ .

Similarly, from equation (3)

$$S_{SP}(d) = \frac{E_1}{I_p M_T} \frac{A \int_0^L E_T(x) e^{-i\psi(x)} dx}{d\lambda \int_0^L E_1 dx}.$$

For a line transducer scanned with a probe transducer over a scan of length  $L$ , from equation (8),

$$M_L = \frac{E_1}{I_T S_T} \frac{L(a\lambda)^{\frac{1}{2}} \int_0^L E_L(x) e^{-i\psi(x)} dx}{d\lambda \int_0^L E_1 dx},$$

and from equation (7),

$$S_{SL}(d) = \frac{E_1}{I_L M_T} \frac{L(a\lambda)^{\frac{1}{2}} \int_0^L E_T e^{-i\psi(x)} dx}{d\lambda \int_0^L E_1 dx}.$$

#### DATA REQUIRED

The near field data required for calibration of piston and line transducers are as follows.

##### Piston Transducer

1. The area scanned (length of line transducer times length of scan).
2. The open-circuit voltage (receiving) of the piston and the phase as functions of position of the line transducer scanning the near field of the piston.
3. The driving current through the line transducer.
4. The transmitting current response of the line transducer.
5. The wavelength of sound in the medium.
6. The frequency.

The open-circuit voltage, phase, and free-field voltage sensitivity of the line transducer and the piston transducer's driving current can replace items 2, 3, and 4 for transmitting response.

The distance from the face of the piston to the scanning plane need not be known, but must be less than  $A/\lambda$ , where  $A$  is the area of the piston source.

### Line Transducer

1. The length of the axis scanned.
2. The open-circuit voltage and phase of the line transducer as a function of the probe position.
3. The current driving the probe.
4. The transmitting current response of the probe transducer.
5. The wavelength of sound in the medium.
6. The frequency.

The open-circuit voltage, phase, and free-field voltage sensitivity of the probe transducer and the line transducer's driving current can replace items 2, 3, and 4 for transmitting response.

The distance  $a$  from the center axis of the line transducer to the traverse axis of the probe transducer must satisfy the relation  $3 < kc < 2\pi(L/\lambda)^2$ .

If a line transducer is used to scan the near field of a piston, the line must be of sufficient length that the sound pressure is down 20 db at the ends relative to the sound pressure at the center of the line. The line transducer must be unshaded and continuous. There should be no standing waves between the piston and the line, and no electrical cross-talk.

Reference [5] shows the degree of uniformity obtainable from an array of piston elements forming a line transducer. This report also shows phase measurements on a steered array and explains by Figure 6 how phase measurements can be used to determine the correction for a depressed beam as discussed under "Data Computation".

### REFERENCES

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